FOUNDATIONS OF MATHEMATICS

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C. W. Kilmister. *Russell.* London: Harvester P.; New York: St. Martin's P., 1984. Pp. x, 252. £40 (paper, £12.95); US$27.50.

This book aims to make accessible to the general reader Russell's early work in logic and foundations of mathematics. The author holds, correctly I think, that understanding this work is essential for comprehension of Russell's more properly philosophical work. A conscientious attempt is thus made to explain mathematical and logical concepts without presupposing substantial background knowledge on the reader's part. The major portion of the book (four of its five chapters) consists of summaries, together with some relevant background and occasional critical discussion, of the following works: *An Essay on the Foundations of Geometry*, *A Critical Exposition of the Philosophy of Leibniz*, "The Logic of Relations: with Some Applications to the Theory of Series", *The Principles of Mathematics*, "On Denoting", "Mathematical Logic as Based on the Theory of Types", *Principia Mathematica*, and the Introduction to the Second Edition of *Principia Mathematica*. Naturally, as the penultimate item assures, these summaries are often highly selective, although it was often unclear to me what the principles of selection were. With the exception of the Russell–Jourdain correspondence, no use is made of material Russell did not publish. (As indicated below, this leads to some serious shortcomings.) While there is fairly extensive, and sometimes uncritical, use of secondary literature, no use was apparently made of the 1980–81 issues of *Synthese* that were devoted to Russell's early work in philosophy and that contain important papers by Cocchiarella, Griffin and Hylton.

Early on in the book, the author identifies two main strands of argument that "run through the whole of the book" (p. 3). The first of these is the contrast he aims to draw between Russell's plan to demonstrate the truth of mathematics and other approaches to the foundations of mathematics. It is surely correct to hold that, as early as *The Principles of Mathematics* (p. 4), Russell sees it as a virtue of logicism that it holds the claims of mathematics to be true. But this was not Russell's view during the years prior to about

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1899. In these early years, he held quite clearly that various parts of mathematics were not even consistent. Indeed a primary point of Chapter IV of *The Foundations of Geometry* was to exhibit the contradictions inherent in basic geometrical notions, such as point. (Significantly, the author offers no substantial summary of this chapter.) Published and unpublished work of this period aims to exhibit contradictions in other mathematical concepts such as quantity, continuity and infinity. Thus Russell's view in the *Principles* is a substantial and important *change* from his early views.

The author tends to assimilate the preceding metaphysical point about the truth of mathematics with the epistemological point that logicism would, if successful, show that we had certain knowledge in mathematics. For example, he writes,

> Russell came to see the need for truth in mathematics as part of his personal need for knowledge in any field. One could almost say that he selected mathematics for the establishment of this certain truth because it was the most hopeful area for it. (P. 3)

> My aim in this book ... is to exhibit this concern for truth as the guiding thought behind the earlier books, as it was for *Principia Mathematica*. The concern can be traced back to his early desire for certainty of knowledge. (P. 168)

But, by the time of publication of *Principia*, Russell certainly did not think that all logical principles were known with certainty, or that the axioms of logic conferred certainty on the arithmetic theorems derived from them. In the Preface to *Principia*, he writes, "... the chief reason in favour of any theory on the principles of mathematics must always be inductive, *i.e.* it must lie in the fact that the theory in question enables us to deduce ordinary mathematics" (*PM* I: v). In his 1907 essay "The Regressive Method of Discovering the Premisses of Mathematics", read before the Cambridge Mathematical Club and first published in Lackey's anthology,[1] Russell argues that the methodology of the logicist project is very much like that of the natural sciences. The method is the hypothetico-deductive method. The ultimate premisses, or logical axioms, are not intrinsically obvious; it is not known that they constitute the only hypothesis that accounts for the "mathematical data"; and thus these general laws "remain merely probable" (p. 274). So while Russell's interest in philosophy of mathematics may originally have stemmed from some early desire for certain knowledge, Russell's own epistemological view about what logicism might accomplish is distinctly fallibilist.

The second main thread of argument that the author sees as running

[1] *Essays in Analyis* (London: Allen and Unwin; New York: Braziller, 1973).

through the book concerns relations. This theme is that "Russell chose to establish the real existence of mathematical entities by using the real existence of relations" (p. 3). This is certainly not wrong, but it fails to bring out what Russell saw as novel and crucial to his views about relations. The novelty is not merely in thinking that relations exist or that they are mind-independent, but rather in denying that all relations are internal; that is, in affirming that there are relations which are purely "external". This comes out clearly at the end of the crucial chapter on Asymmetrical Relations in *The Principles of Mathematics*. There he writes,

> We have now seen that asymmetrical relations are unintelligible on both the usual theories of relation. Hence, since such relations are involved in Number, Quantity, Order, Space, Time, and Motion, we can hardly hope for a satisfactory philosophy of Mathematics as long as we adhere to the view that no relation can be "purely external." (P. 226)

In the final chapter of *Russell's Idealist Apprenticeship*, Griffin argues persuasively, and in great detail, that Russell came to see many of the mathematical "antinomies" that pervaded his earliest work as all having their roots in the assumption that relations were invariably internal. This crucial issue is mentioned only twice in the book, and then only in a cursory way (pp. 42–3, 78).

There are a variety of inaccuracies and infelicities that greatly diminish the potential usefulness of the book. Here are some examples.

On page 89, the author claims that Russell did not believe the Russell Paradox to be "a very important matter until he had written to Frege about it and received his reply...." However, the copy of Parts I and II of *The Principles of Mathematics*, sent to the printer in May 1902, prior to Russell's correspondence with Frege, argues vigorously for the gravity and generality of the paradoxes.[2]

On page 193 of the book, the author quotes the Schröder–Bernstein theorem, as symbolized and proved in ∗73 of *Principia*. He then comments: "This is perhaps more transparent in the form which is now more usual; for any two classes α, β either α is similar to a sub-class of β or β is similar to a sub-class of α or both." This is wrong. The latter is the Principle of Trichotomy, an equivalent of the Axiom of Choice, whereas the Schröder–Bernstein theorem is provable without the Axiom of Choice, as the

[2] For clear evidence, read the text of Chapters X and XV against the printer's copy, presented for Part I, by Blackwell (*Russell*, n.s. 4 [1984]: 271–88) and for Part II, by Byrd (*Russell*, n.s. 7 [1987]: 60–70).

proofs at *73 and *94 show. Russell explains this matter at the beginning of *117.

On page 238, the author interprets Gödel's Second Theorem as implying that the consistency of a formal system "can only be proved in a *more* complex system, whose own consistency is in greater doubt." Gödel's Second Theorem implies only that the system in which consistency is proved deploy some methods not contained in the system to be proved consistent. The system in which consistency is proved may otherwise be much weaker, as Gentzen's proof of the consistency of formalized arithmetic shows.

Contrary to page 209, Russell did not begin *The Philosophy of Logical Atomism* in prison in 1918. The lectures were given for eight weeks in January, February and March of 1918, prior to Russell's imprisonment. *The Philosophy of Logical Atomism* is a set of verbatim reports of the lectures, taken at the time by a shorthand writer. (See *Collected Papers* 8: 157.)

Often quotations are footnoted only by the book or article in which they occur, not by page number. To take a limited sample, this is done on pages 61, 63, 64, 65, 66, 70 and 72.