"ON DENOTING" AGAINST DENOTING

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I. INTRODUCTION

In his article "On Denoting", Russell infamously announced that the whole theory of meaning and denotation is enveloped in an "inextricable tangle" and has to be abandoned. The "tangle", however, has yet to be recovered from its expression in the obscure "Gray's *Elegy* argument" of the work—but not for lack of trying. Indeed, there are so many different interpretations of the argument now in the literature that we do well to ask by what criteria shall any account of the argument be assessed. Is there any way that the real Gray's *Elegy* argument could be recognized?

Years ago Geach (*1958*) suggested an answer. The argument is best understood if targeted at the theory of denoting of Russell's 1903 *Principles of Mathematics.* Cassin (*1970*) agreed, and attempted to set out the argument within the historical context of the *Principles.* It is no longer possible to doubt the correctness of this approach; Russell himself unequivocally authenticates it a June 1905 manuscript entitled "On Fundamentals". Russell even scrawled on the first leaf that pages 18ff. contain the reasons for the new theory of denoting. The manuscript arrives at the "inextricable tangle" which, for the most part, would be presented in "On Denoting"; and the working notes proceed wholly independently of Frege, by working though difficulties with the *Principles'* theory of denoting.

The Gray's *Elegy* argument has proved to be resistant, however. It is a siren song which lives up to the *Odyssey* in bringing to ruin all who hope to probe its pages. The manuscripts Russell left unpublished[1] are as obscure and involuted as ever they could be; and new possibilities for reading his many animadversions grow exponentially. One must stay the course, holding onto what Russell regarded as fundamental principles. But what principles were fundamental? Sooner or later the interpreter is seduced by one or another of the songs and, impelled by its blissful soporific, dashed on the rocky crags.

This paper takes the lead of Odysseus, navigating past the sirens by being firmly lashed to the mast of the *Principles*. The firm lashing was generated by my work on Russell's so-called "substitutional theory of classes and relations" which he revamped in December 1905 under the auspices of the theory of definite and indefinite descriptions. Russell regarded this theory as the natural ally of the *Principles*, adhering, as he put it, with "drastic pedantry", to the fundamental doctrine of the *Principles* that "whatever is, is one" ([STCR], p. 189). With the historical development and mechanics of the substitutional theory understood, and the security of Russell's explicit view that it adheres to the *Principles'* fundamentals, we can resist the siren songs of Russell's manuscripts. In a letter to Jourdain of 14 March 1906, Russell recounted the road to substitution. He wrote:

> About June 1904, I tried hard to construct a substitutional theory more or less like my present theory. But I failed for want of the theory of denoting: also I did not distinguish between *substitution* of a constant for a constant and *determination* of a variable as this or that constant....
>
> Then, last autumn, as a consequence of the new theory of denoting, I found at last that substitution would work, and all went swimmingly.[2]

The substitutional theory emerged from Russell's attempt (in the *Principles*) to use denoting concepts and the notion of the substitution of entities (including denoting concepts themselves) in the explanation of the constituents of propositions named by formulas involving the use of single letters as variables. The problem of the logical form of the propositions in question is thrust to the fore; and, as we shall see, this reveals

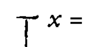
[1] These include "Points about Denoting", "On the Meaning and Denotation of Phrases", "On Meaning and Denotation", and "On Fundamentals."

[2] Grattan-Guinness, *1977*, p. 79–80.

the sound argument Russell had against denoting concepts.

2. FORMAL IMPLICATION

The introduction of variables into a symbolic calculus for Logic was an invention originating with Frege's 1879 *Begriffsschrift*. Borrowing the arithmetic notion of a function he invented a function calculus of "truth-functions". He has:



$$= \begin{cases} \text{the False, if } y \text{ is the True and } x \text{ is any object other than the True} \\ \text{the True, otherwise} \end{cases}$$

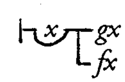
$$\top x = \begin{cases} \text{the True, if } x \text{ is any object other than the True} \\ \text{the False, otherwise} \end{cases}$$

A natural language sentence is then transcribed into the function calculus by assigning function letters. For instance, put:

$$fx = \begin{cases} \text{the True, if } x \text{ is human} \\ \text{the False, otherwise.} \end{cases}$$

$$gx = \begin{cases} \text{the True, if } x \text{ is mortal.} \\ \text{the False, otherwise.} \end{cases}$$

Then using "⊢" to transform a term for a truth-value into a sentence, Frege writes,



to say that all men are mortal. This models predication in terms of mathematical functionality. It does so by abandoning from the onset all vestiges of the Aristotelian (and medieval) theory of categoricals—a theory which regarded the linguistic fact that "all men" is the subject expression of the sentence "All men are mortal" as of *logical* significance.

Geach (*1962*) regards Aristotelian and medieval treatments of quantification, and indeed any treatment that takes quantified noun phrases such as "all men", "some men", "a man", and so on, as syntactic and semantic units, a "shipwreck of a theory". Frege's introduction of variables into the calculus for logic was a substantive advance. Indeed, with it began the quest for "logical form", as opposed to linguistic grammatical form. Frege's logicism depended upon this, for by its means he hoped to show that ordinary arithmetic statements have a deep logical structure that grounds their truth. It was their superficial surface grammar that misled Kant into rejecting them as theses of pure logic.

By 1900 Russell himself had become aware of the fact that the introduction of variables into a calculus for logic enables a "logic of relations" which, advancing well beyond Aristotelian and medieval schools, can reach arithmetic. He learned of the introduction of variables from the work of Peano and his school, at a fateful congress held in Paris in 1900. Shortly thereafter, Russell rediscovered Frege's account of cardinal number and arrived at logicism.

In the *Principles*, Russell espouses the view that logic is an all-encompassing, wholly universal science which applies to all entities whatsoever. Logic, he maintains is the synthetic *a priori* science of structure. The structures that are the subject-matter of the science of Logic were reified as "propositions". The fundamental doctrine of the *Principles* is *Quodlibet ens est unum*—"Whatever is, is one" (*PoM*, p. 132). That is, every entity can occur in a proposition "as one", e.g., the way Socrates (the man) occurs in the proposition,

'Socrates is wise'.[3]

And in so far as every entity occurs "as one", Russell maintained that any calculus for the science of logic should adopt only one style of variables —viz., "entity" variables.

Russell modelled his calculus for logic on Peano's work. Peano did not devise a "function calculus" as Frege had, but a sentential calculus, introducing as sentential forms *implication*,

$$\alpha \supset \beta$$

[3] I shall use single quotation marks for ontological entities such as propositions or denoting concepts. Double quotes, even when multiply embedded, will be used for linguistic strings.

and *formal implication*,

$$\alpha \supset_{x_1, \ldots, x_n} \beta,$$

where α and β are any terms. Here "$\supset$" stands for the dyadic relation 'implication' and analogously "$\supset_{x_1, \ldots, x_n}$", stands for the relation of formal implication.[4] Russell implicitly adopted Peano's implication as a primitive notion, but Peano's formal implication wanted philosophical analysis. First, Peano (by his own admission) found the rules for deduction with formal implication "abstruse" (Peano *1901*, §18). Of course, unlike Frege, Peano allowed conditional proof and generalization under an assumption.[5] His difficulties with deduction with formal implication derive from his uncertainty as to the conditions under which such generalization should be legitimate. Russell wanted to correct and amend Peano's techniques—spelling out the proper transformation rules and axioms of his calculus for logic. An analysis of formal implication should show the way. The second reason Russell wanted an analysis of "formal implication" was that his calculus allows that any well-formed formula of its language can be nominalized to form a genuine name of a structure (a general proposition). This confronts the question as to what are to be the constituents of those propositions named by formal implications.

In the *Principles*, Russell's philosophical analysis was his theory of denoting. According to this view, "all *a*", "every *a*", "some *a*", "no *a*", and "the *a*", etc., stand for entities called "denoting concepts". Such concepts occur in propositions as constituents. In the proposition indicated by the nominalized sentence "All men are mortal", the denoting concept 'all men' occurs. The proposition is about all men. The idea is of a piece with the Aristotelian and medieval theory of categoricals, and Russell even employed a form of the medieval distinction between *suppositio determinata* and *suppositio confusa* to get at differences in scope.[6] The theory of denoting was to form a conceptual bridge from the Aristotelian (and medieval) treatment of categoricals to the use of single letters as variables in the expressions Peano called formal implications.

[4] Hereafter I shall call any well-formed formula that has one or more variables bound to an initial quantifer a "formal implication".
[5] See Landini *1996*.
[6] See *PoM*, p. 57. Compare Geach *1962*.

With this theory, Russell hoped to answer questions concerning the constituents of the propositions indicated by nominalization of formal implications. That is, the theory was to analyze and legitimate the use of single letters as variables and the notion of the assignment ("determination") of a variable at this or that value.[7]

3. THE *Principles*' ANALYSIS

Formal implication involves the use of single letters as variables. Russell took the idea of variation literally in his analysis of the constituents of those propositions indicated by nominalizations of sentences involving variables. The analysis appealed to denoting concepts and the auxiliary notion that given a proposition such as,

'*a*'s being human implies *a*'s being mortal',

we can collect together all propositions like it except those having different entities occurring wherever *a* occurs (*PoM*, p. 38). Russell speaks of substituting, say, the entity *b* for *a* in the proposition

'*a*'s being human implies *a*'s being mortal'.

The nominalization of the sentence ("formal implication"),

"*x*'s being human implies, for all *x*, *x*'s being mortal"

indicates (roughly) the following proposition:

'every entity of the class of entities just like '*a*'s being a man implies *a*'s being mortal' except having any entity at the position of *a* is true'.

What of the use of predicate variables? According to the *Principles*, the use of predicate variables would be introduced via definitions. Consider the proposition,

'*a* differs from *b*'.

[7] See *PoM*, p. 39.

We want to render an analysis of the constituents of the proposition indicated by the nominalization of the sentence,

"$(\exists\phi)a\phi b$",

which uses "ϕ" as a predicate variable. Russell notes that the following would not work:

Some entity (proposition) of the class of entities (propositions) exactly like '*a* differs from *b*' except containing any entity at the position of 'differs from' is true.

The trouble is that it is a violation of structure to substitute, say, Socrates for 'differs from' in the proposition '*a* differs from *b*'. To solve this problem,[8] Russell thinks that the formal system must have primitive predicate constants, $C^n(x)$ for the property of being an n-place concept or relation, and predicate constants $E^{n+1}(x_1, \ldots, x_n, y)$ for relations of exemplification. Accordingly,

$$(\exists\phi)a\phi b =_{\mathrm{df}} (\exists y)(C^2(y) \;\&\; E^3(a, b, y)).$$

The proposition indicated by a nominalization of the formula would then be something like:

'Some entity (proposition) of the class of all entities (propositions) exactly like ''differs from' is a concept exemplified by *a* and *b*' except containing any entity at the position of 'differs from' is true.'

Russell's analysis has many difficulties. But we can at least see how he imagined that the theory of denoting concepts could be employed in the analysis of the constituents.

It is very important to realize that the *Principles*' analysis does not "ontologize variables" by assuming an ontology of propositional functions and absorbing the variable into them.[9] Russell seeks an analysis of

[8] See *PoM*, p. 86, for Russell's first stab at solving this problem. He returns to the problem in the manuscript "On Fundamentals" as we shall see.

[9] The view that Russell ontologized variables as part of his theory of propositional functions has been recently advocated in Hylton *1990*.

the constituents of propositions expressed by wffs containing variables. His analysis does *not* assume that every open wff comprehends a Platonic entity—a "propositional function". We must not be misled by Russell's writing that "... it appears that propositional functions must be accepted as ultimate data" (*PoM*, p. 88). This comment comes when Russell makes the point that the use of variables cannot be explained by any simple use of the denoting concept 'any entity'. In the expression,

"x's being a man implies x's being mortal"

we cannot get at the way the variable is assigned values by taking the "assertional form",

...'s being a man implies ...'s being mortal.

The use of the letter "x" marks a sameness of reference in the original and this is lost in above assertional form (*PoM*, p. 85). An instance of this form could be the proposition

'Socrates's being a man implies Plato's being mortal'.

Thus, Russell concludes that "likeness of form" must be a primitive notion unanalyzable (except perhaps in a simple subject–predicate proposition) by means of separation of a proposition into subject(s) and assertional form. Russell's analysis of formal implication requires that we can group by "likeness of form" all propositions got by substitution. But Russell explicitly warns that what is primitive is not particular propositional functions, but the *class concept* 'propositional function' (i.e., likeness of form) (*PoM*, p. 92). Indeed, the fact that the use of (predicate) variables is to be analyzed away in terms of substitution and denoting is not incidental to Russell's hopes of finding a solution of the paradoxes (of predication) plaguing logicism. To be sure, a purely formal dodge which avoids the paradoxes was sketched in the *Principles*' Appendix B. This was to stratify classes into types. Russell knew, however, that stratification is at odds with the fundamental doctrine that a calculus for logic must adopt only one style of variables. If classes are single entities, then nothing can prevent "$x\varepsilon x$" from being meaningful. Nonetheless, Russell expressed hope that by means of the theory of denoting concepts

the stratification could be built into the logical grammar of a no-classes-as-one theory. Classes are not single entities, but perhaps the theory of denoting concepts might generate a theory of "plural logical subjects" and thereby show how a proxy for a theory of classes-as-one could be generated (*PoM*, p. 516f.). The *Principles* had no such theory and did not adopt types; rather it sought axioms which excluded "quadratic forms" so as to determine when it was safe to assume a class as a single entity.

"Quadratic forms" are sentential forms where, beginning from a formula $A\mu$, one introduces variables "ϕ" and "x", and represents the structure of the proposition (named by a nominalization of the formula) by "ϕx". Russell thinks that quadratic forms will be excluded by a proper analysis of the use of single letters as variables. With characteristic sloppiness of use and mention, Russell proclaims that it is not (always) possible to vary ϕ independently of x in ϕx.[10] Of course, the official view of the *Principles* was that predicate variables such as "ϕ" would only be introduced into the calculus via definitions wrought from the analysis of the use of variables that denoting concepts provided. The analysis should yield when it is legitimate to introduce a predicate variable. The analysis was nowhere complete or satisfactory to Russell. But the preliminary conclusion of the *Principles* was that quadratic forms will be excluded and their exclusion will block the paradoxes of predication and classes that plague logicism. As Russell puts it:

> ... according to the theory of propositional functions here advocated, the ϕ in ϕx is not a separable and distinguishable entity: it lives in the propositions of the form ϕx, and cannot survive analysis. (*PoM*, p. 88)

Of course, to reach arithmetic via the theory of classes (and relations-in-extension) one often takes a formula $A\mu$ to be representable by "ϕx", with "ϕ" and "x" variables. So Russell's analysis of the use of variables by means of the notion of denoting and the substitution of entities in propositions had to find a way to filter out the innocuous cases of such variation. Labour as he might, Russell failed to discover any filtering principles that could count as being principles of logic. If a solution can be found, Russell decided, it will be found in the analysis of general propositions.

[10] See [FN], *Papers* 4: 142; and [OMD], *Papers* 4: 348.

4. THE CRUX OF THE PROBLEM WITH DENOTING CONCEPTS

As we see, since the use of single letters as variables is to be analyzed in terms of denoting and substitution, the sort of occurrence of an entity in a proposition—an occurrence for which other entities are to be substituted—is of central concern. It is not even possible for Socrates to occupy the position of the concept 'humanity' in the proposition 'Plato is human'. There is a fundamental difference in structure here. Substituting one entity for another will capture the use of entity variables only if the entity which is the substitute occurs in the proposition "as one", rather than predicatively.

The fundamental doctrine of the *Principles* is that every entity can occur "as one" in a proposition and that, accordingly, any calculus for logic should adopt only entity variables. The calculus treats all entities, be they propositions, concrete particulars, universals, or whatever, alike. Russell calls an occurrence of an entity "as one" in a proposition an occurrence as "logical subject". The notion of a logical subject is a primitive of the *Principles*. The phrase "logical subject" is used synonymously with "term", "entity", "individual", "being" and "one" (*PoM*, p. 43). To be sure, the concept 'humanity' is an entity, and so a logical subject. But it does not occur "as logical subject" in the proposition 'Socrates is human'. Russell writes:

> I shall speak of the *terms* of a proposition as those terms, however numerous, which occur in a proposition and may be regarded as subjects about which the proposition is. It is a characteristic of terms of a proposition that any one of them may be replaced by any other entity without our ceasing to have a proposition. (*PoM*, p. 45)

An entity occurs "as logical subject" (or alternatively "as term of a proposition") when it is both a constituent of a proposition and what the proposition is about (*ibid.*).

As we saw, the unrestricted nature of the variable is captured in the fundamental doctrine of the *Principles*—viz., "whatever is, is one". Otherwise put, this says that every entity may occur in a proposition as a logical subject. The doctrine is grounded in what Russell calls the "indefinable two-fold nature" of concepts. In the *Principles*, Russell divided all entities into two sorts: "things" and "concepts" (p. 44). The division is based on the fact that concepts can occur in a proposition "as concept" (i.e., "predicatively" in the case of properties and relations) and can occur in a proposition as "logical subject" or, as Russell sometimes called it, "as term of a proposition" (*PoM*, p. 45). *Things* are different from concepts in that they can *only* occur as logical subjects (*ibid.*). Moreover, it is the very self-same concept that occurs "as term" or "as concept". Russell agreed with Benno Kerry that the sentence "the concept horse is not a concept" is self-refuting. Thus he would have nothing of Frege's notion of the essential "unsaturatedness" of concepts (*ibid.*, pp. 46 and 505).

Indeed, Russell could not accept Frege's modelling of predication by mathematical functionality. To Russell, it is propositions and the ways entities occur in them that are primitive, not the mathematical notion of a function carrying its "argument" to its value. Occurring as logical subject (or "as term") is not to occur as "argument" to a Fregean function. For Frege, when a function takes its argument, there is no whole composed of function and argument.[10] *For Russell an entity has a property or stands in a relation only in so far as it occurs as logical subject of a true proposition predicating the property or relation.*

What of unity? For Russell it is the occurrence of a property or relation "as concept" that accounts for the unity of a proposition. For Frege, the unity of a *Gedanke* is explained by the essentially unsaturated nature of the sense of a predicate expression. But Frege does not model the sense as a function. A *Gedanke* is a whole composed of the senses of the significant parts of a sentence. The sense of "John" does not occur in the *Gedanke* expressed by "John's being tall" as argument to a function.[11] It certainly does not occur in a way analogous to Russell's notion of "occurring as term". Its closest analog to Russell is an occurrence "as concept".

Returning to Russell, we found that it is because of the twofold occurrence of concepts that the fundamental doctrine of the *Principles* can be maintained. Concepts can occur "as term" and thereby are values of the single style of variable—the individual variable. But if the twofold occurrence of concepts is central to the fundamental doctrine of the *Principles*, then what of denoting concepts? Do denoting concepts have

[10] I follow Dummett *1973*, p. 267, in this view.
[11] See Dummett *1973*, p. 179.

the twofold nature? The only answer possible in the context of the *Principles* is "yes". For to be a term (logical subject) is to be capable of occurring "as term" in a proposition.

In the manuscript "On Fundamentals", Russell replaced the terminology "occurring as concept" with "occurring as meaning", and replaced "occurring as term" (or "as logical subject") with "occurring as entity" (*Papers* 4: 369ff.). The fundamental doctrine of the *Principles* can now be expressed as: "Whatever is, must be able to occur in a proposition as entity". Properties and relations have a twofold nature; they can occur "as entity" and "as meaning" (predicatively). To illustrate, the denoting concept,

'the author of *The Principles of Mathematics*'

occurs *as meaning* in the proposition

'The author of *The Principles of Mathematics* is wise',

for the proposition is about Russell. Similarly, the denoting concept occurs *as meaning* in the complex denoting concept,

'the godfather of the author of *The Principles of Mathematics*'

for this denoting concept denotes Mill.

Russell discovered the problem of capturing the difference in structure in his attempt to analyze, by means of denoting and substitution, the constituents of propositions named by nominalizations of sentences employing single letters as variables. As we saw, the analysis was to reveal the nature of functionality, and to sort out the jumble of notions Russell termed under the problem of "propositional functionality".

In the manuscripts of 1903, two notations appear. In "Points about Denoting" and "On the Meaning and Denotation of Phrases" we find:

$$p\frac{x}{y}$$

This is used to proxy at mathematical functionality, viz., $f(x) = y$. There is also the matter of "ϕx". In "On Fundamentals" we find Russell with the notation "$(C)(\frac{z}{\text{var}})$". This seems to be a variant of the earlier, but now "C" is used to pick out denoting concepts or propositions. Just as in the *Principles*, the "illegitimacy of variation of ϕ separately from x" is to be explained by the analysis of the variable via denoting; and as before, this was to block the paradox of predication.[13] In "On Fundamentals" Russell puts it as follows: "... what occurs as meaning can't be varied; we must be able to specify what varies, and this can only be done if what varies occurs as entity, not as meaning" (*Papers* 4: 362). But a proxy for such variation in innocuous cases has to be found. The plan was now to find it in the theory of denoting concepts. Where one wanted to vary "ϕ" separately from "x" the denoting phrase "$(C)(\frac{z}{\text{var}})$" was to be used (*ibid.*, 4: 396).

There are scant remarks concerning the construction of proxy for class expression via denoting concepts. But it is not difficult to imagine Russell substituting denoting concepts for one another to render a proxy. To proxy,

$$\hat{z}(z \text{ is a man})\ \varepsilon\ \hat{a}\{a = \hat{z}(z \text{ is a mortal})\}$$

Russell might have contemplated putting something like:

$$\{(z)(z \text{ is an animal} .\supset. z \text{ is a mortal})\}\ (\frac{\text{'a man'}}{\text{'an animal'}}).$$

But to go very far he will need to use "$(C)(\frac{z}{\text{var}})$". For instance to proxy the wff,

$$a\ \varepsilon\ U\hat{a}\{a = \hat{z}(z \text{ is a mortal})\}$$

Russell will need:

$$\{(\exists C)((z)((C)(\frac{z}{\text{var}}) .\supset. z \text{ is a mortal}) .\&. (C)(\frac{x}{\text{var}}))\}\{\frac{a}{x}\}$$

Though there is little about classes, "On Fundamentals" gives some attempted analyses of the paradox of predication (*Papers* 4: 364). Early on, we find the following sort of discussion:[14]

[13] See *PoM*, p. 88.

[14] I have modified this somewhat from the original for ease of presentation.

$$(R)(\frac{z}{C}) =_{\text{df}} \{\sim(C)(\frac{C}{\text{var}})\}(\frac{z}{C})$$

Here, "var." is used to pick out "the variable" in C.[15] Then we are to suppose that

$$(R)(\frac{(R)(\frac{z}{C})}{C}).$$

Performing the substitution, we get,

$$\sim((R)(\frac{z}{C}))(\frac{(R)(\frac{z}{C})}{\text{var}})$$

We then arrive at:

$$\sim(R)(\frac{(R)(\frac{z}{C})}{C})$$

But performing the relevant substitutions again, we are back to

$$(R)(\frac{(R)(\frac{z}{C})}{C}).$$

This is the paradox of predicates generated via denoting and substitution.

The 1903 manuscripts, "Points about Denoting" and "On Meaning and Denotation", as well as the 1905 "On Fundamentals", show Russell agonizing over the meaningfulness of his notation. In "Points about Denoting" he offers the following remark:

> "$p/\frac{x}{y}$" means "that which is denoted by the meaning which results from giving x the value y in p". (*Papers* 4: 308)

Let p be the phrase "the godfather of any term". Consider then the denoting concept indicated by the phrase "$p/\frac{x}{y}$", viz.,

[15] We can avoid the use of "var" by putting:

$$(R)(\frac{z}{C}) =_{\text{df}} \{\sim(C)(\frac{C}{z})\}(\frac{z}{C})$$

But note that $(R)(\frac{(R)(\frac{z}{C})}{C})$ is to be $\{\sim(C)(\frac{C}{z})\}(\frac{(R)(\frac{z}{C})}{C})$.

> 'that which is denoted by the meaning which results from giving any term the value of any term in the godfather of any term'.

This is fraught with difficulties. The denoting concept 'any term' has three occurrences here. Each of the three are meaning occurrences and so each might denote a different entity. Evidently, Russell though of 'any term' as occurring as entity in the second and occurring as meaning in the others. Correction gives,

> 'that which is denoted by the meaning which results from substituting any term for 'any term' in the godfather of any term'.

But even with this correction, the denoting concept 'the godfather of any term' has a meaning occurrence, and so denotes the godfather of some term. Suppose it is Russell. We are enjoined to substitute some term, say Frege, for 'any term' in Mill!

Russell was soon to detect these flaws. In "On Meaning and Denotation" he observes that his former reading of "$p/x;y$" requires correction:

> ... if we take any single letter p, a particular value of p may well be the value of some complex containing x for a particular value of x, but this cannot be the complex with x still variable. And this shows that p or $p/x;y$ won't do, since it forgets that a variable x is not in any of its values.... (*Papers* 4: 336)

Let us then modify the above as follows:

> 'that which is denoted by the meaning which results from substituting any term for 'any term' in 'the godfather of any term'.'

Then letting the meaning occurrence of 'any term' pick out Frege, the result is the denoting concept 'the godfather of Frege' and the whole denotes (if anyone) Frege's godfather. But our troubles are not over. We have violated the structure of the denoting concept 'the godfather of any term'. We substituted Frege for a meaning occurrence of 'any term'. In "Points about Denoting" and more saliently in "On Fundamentals", Russell exacts an additional point against his former rendering of "$p/x;y$". He puts:

p = the present Prime Minister of England
q = the nephew of the later Prime Minister of England
x = England
y = France

He then observes that

$$p = q \text{ .\&. } p/x;y \neq q/x;y.$$

This, he says, shows that his former reading will not do: "If p stands for the complex meaning, $p/x;y$ should stand also for a complex meaning, not for what this [meaning] denotes" ([PAD], *Papers* 4: 309). That is, if the substitution of France for England is to take place in the denoting concept p, then the denoting concept '$p/x;y$' should denote the denoting concept 'the present Prime Minister of France' and not the present Prime Minister of France. Yet it is the Prime Minister that is wanted as the denotation. Russell is labouring to capture functionality here. In the functional $fx = y$, the value y is not composed of function and argument at all. But in substitution one does not arrive at y, but a complex denoting concept.

It was the attempt to analyze variation by means of denoting concepts and substitution that directed Russell to a new discovery—viz., the impossibility of a theory of logical form which would ground the structural difference between the logical form of a proposition (or complex denoting concept) in which a denoting concept occurs *as meaning* and the quite different logical form of a proposition (or complex denoting concept) in which a denoting concept occurs *as entity.*

Some years ago, Cassin may have glimpsed this. Consider the following criticism of Cassin by Blackburn and Code. Blackburn and Code hope to reject the context of the *Principles* and redirect Russell's argument to Frege. They write:

> Authors have not failed to invent features of Russell's earlier view, so that his argument can be seen as relevant to them, but not to Frege. Cassin ... holds that it was part of his [Russell's] earlier view ... that "only terms could be denoted" but now Russell has come to realize that denoting concepts themselves must be denoted. This is just wrong about the earlier view. A term in the *PoM* is *anything* which can be the subject of a proposition, or an object of thought, or a logical subject (§47) and both things *and* concepts are terms (§48). (Blackburn and Code *1978*, p. 68)

To be sure, all entities are terms for Russell; and terms must include denoting concepts. But Blackburn and Code miss the problem of structure entirely. They write:

> ... the *PoM* theory of denoting makes it impossible to directly name a sense or denoting concept. This is because (i) when a name is used in a sentence, the thing named is a constituent of the proposition expressed by the sentence, and (ii) if a denoting concept is a constituent of a proposition, then the proposition is about the object denoted and not about the denoting concept. If we suppose that he had come to see this by the time he wrote "On Denoting", we have an explanation for the fact that he is now insisting that senses be introduced by means of definite descriptions. (*Ibid.*, p. 76)

This is in error. Russell does not say that *whenever* a denoting concept occurs in a proposition, the proposition is about the denotation of the concept (if any). He says that a concept denotes when, "if it occurs in a proposition, the proposition is not *about* the concept, but about a term connected in a certain peculiar way with the concept" (*PoM*, p. 53). It is only when a denoting concept occurs "as concept" in a proposition that the proposition is about its denotatum. Nothing here excludes the occurrence of a denoting concept as "term of a proposition". If it did, the fundamental doctrine of the *Principles* would be lost. As we have lately seen, all terms (logical subjects), including denoting concepts, must occur in propositions "as term of the proposition". The crux of the problem, as Cassin seems to see (albeit opaquely) is to explain how.

5. THE GRAY'S ELEGY ARGUMENT

Let us now look at Russell's "Gray's *Elegy* argument" from "On Denoting". Russell tells us that, when a denoting phrase such as "the first line of Gray's *Elegy*" is in a grammatical subject position of a sentence, the denoting concept indicated by the phrase occurs "as meaning" ("as concept") in the proposition indicated by a nominalization of the sentence. This is the case in the proposition indicated by a nominalization of the sentence,

(1) "The first line of Gray's *Elegy* states a proposition."

Here the denoting concept 'the first line of Gray's *Elegy*' occurs "as meaning". Compare the sentence,

(2) " 'The first line of Gray's *Elegy*' does not state a proposition."

In the proposition indicated by a nominalization of this sentence, the denoting concept 'the first line of Gray's *Elegy*' was to have occurred "as entity" (i.e., "as term of the proposition").

The case is similar when a denoting phrase occurs as grammatical subject of another phrase. For instance, the denoting concept 'the first line of Gray's *Elegy*' occurs "as meaning" in the complex denoting concept indicated by the phrase,

"the meaning of the first line of Gray's *Elegy*".

This complex denoting concept denotes the meaning of "The curfew tolls the knell of parting day". The same holds when we put a denoting phrase in the grammatical subject position of "the denotation of …". Follow Russell in putting the phrase "the denoting complex occurring [as entity] in the second of the above instances" in the dots. (The second instance is (2) above.) The result is

"the denotation of the denoting complex occurring in the second of the above instances".

Use of this phrase speaks about the denotation of the denoting concept

'the first line of Gray's *Elegy*',

viz., "The curfew tolls the knell of parting day".

Russell concludes that to get what is wanted—i.e., to denote the denoting concept 'the first line of Gray's *Elegy*'—we must employ

"The meaning of "the first line of Gray's *Elegy*"."

or employ

"The denotation of "the denoting concept occurring in the second of the above instances"."

So far this is mundane. What does Russell think himself to have accomplished?

Russell thinks himself to have demonstrated that since grammatical subject positions indicate "meaning occurrences" there can be no entity occurrences of denoting concepts! This is wrong. Russell has conflated the linguistic notion of a grammatical subject position with the ontological notion of an entity occurrence.

The same error occurs in Russell's manuscript "On Fundamentals". He writes:

The use of inverted commas may be explained as follows. When a concept has meaning and denotation, if we wish to say anything about the meaning, we must put it in an *entity*-position; but if we put it itself in an entity-position, we shall be really speaking about the denotation, not the meaning, for that is always the case when a denoting complex is put in an entity-position. Thus in order to speak about the meaning, we must substitute for the meaning something which *denotes* the meaning. (*Papers* 4: 381–2)

This is sloppy. Russell speaks of what he calls an "entity position". Properly understood, an entity position is just the notion of being a grammatical subject of a larger phrase. Unfortunately, he sometimes conflates this notion with the ontological notion of an occurrence in a complex. A denoting concept does not occur in an entity position (grammatical subject position). It is the denoting phrase that is in entity position. Russell has not shown that entity occurrences are impossible.

In light of this, one might be tempted to seek a better fit with Russell's writings and conclude that our identification of the term/concept distinction of the *Principles* with the "entity/meaning" distinction of "On Fundamentals" is mistaken. Consider Russell's "broad rule" of "On Fundamentals":

… when complexes occur as *meaning*, their complexity is essential, and their constituents are constituents of any complex containing the said complexes; but when complexes occur as *entities*, their unity is what is essential, and they are not to be split into constituents. Hence generally: When a complex *A* occurs in a complex *B*, if *A* occurs as *meaning*, its constituents are constituents of *B*, but if it occurs as *entity*, its constituents are not constituents of *B*. (*Papers* 4: 373)

Russell decides that when a complex denoting concept occurs as entity in a larger complex it is a unity, and is considered as "one" constituent of the larger. When it occurs as meaning its unity is lost. For instance, when the concept 'the author of *Waverley*' occurs as meaning it is the indication of the words "the", "author of", and "*Waverley*" that are among the constituents of the larger complex.[15] In this case, the denoting concept cannot be regarded as a single constituent of the larger.

Russell is here struggling to understand the difference in logical form between occurrence as entity (term) and as meaning (concept) as applied to denoting concepts. But he recognizes that the "broad rule" is too strong in requiring that the constituents of an entity A occurring "as entity" in B do not themselves occur "as entity" in B. Since implication is a relation, Russell observes that the proposition p occurs in '$p \supset q$' as entity and, thereby,

$$(x)(x \supset q)$$

is intelligible. But now p also occurs "as entity" in the proposition,

$$\text{‘}p \supset q :\supset: q \supset r .\supset. p \supset r\text{’}$$

(i.e., complex B) in spite of the fact that '$p \supset q$' (i.e., complex A) as well as '$q \supset r .\supset. p \supset r$' occur as entity in the whole and p is a constituent of these. So the broad rule would inappropriately exclude

$$(x)(x \supset q :\supset: q \supset r .\supset. x \supset r).$$

Shortly after articulating the broad rule, Russell imagines that he is "forced to recognize a greater variety of modes of occurring than we have yet imagined". He writes in "On Fundamentals":

> We shall have to say that in "Scott is the author of *Waverley*", "the author of *Waverley*" occurs as entity in two senses: (1) any other entity, simple or complex, may be substituted without loss of significance; (2) the denotation of "the author of *Waverley*", or any other complex with the same denotation, may be substituted without altering the truth-value of the proposition. (*Papers* 4: 373)

[15] This view also occurs in "Points about Denoting", *Papers* 4: 306.

This makes it appear as though the entity/meaning distinction is far more complex than the simple term/concept distinction of the *Principles*.

Readers beware! These "other modes" are but siren songs! They distract readers from the fact that Russell's problem is to explain how denoting concepts could have the "two-fold occurrence" (as Russell calls it in the *Principles*) enjoyed by concepts generally. The simple fact is that Russell has wrongly dismissed entity occurrences of denoting concepts by conflating the ontological "entity occurrence" with the linguistic notion of an "entity position". The ontological issue must be separated from the linguistic. The linguistic replacement of expressions does not violate ordinary grammar. We can replace "the author of *Waverley*" in the sentence

"The author of *Waverley* is famous"

by "Scott" without doing violence to ordinary grammar. But ordinary grammar, as Russell was soon to reveal in his famous discovery, is often misleading as to ontological structure. The ontological substitution of the entity Scott, for the denoting concept

'the author of *Waverley*'

in the proposition

'The author of *Waverley* is Scott'

does violate ontological structure.[16] The denoting concept 'the author of *Waverley*' does not occur in the proposition "as entity" (i.e., as term of the proposition). If it did, the proposition would be about the denoting concept itself!

The quest for a theory which fits Russell's discussions of "a greater variety of modes of occurring" is misguided. Russell's conflation of "entity occurrences" with "entity positions" does not vitiate his argu-

[16] This point is echoed in "On Denoting", when Russell observes that Leibniz's Law cannot apply to the proposition indicated by "the author of *Waverley* is Scott" because the phrase "the author of *Waverley*" does not indicate one entity occurring in the proposition in a position for which Scott could be substituted ([OD], p. 114; *Papers* 4: 423).

ment. Russell requires, and yet fails to have, a theory of logical form that captures the difference in structure between meaning and entity occurrences of denoting concepts. Since meaning occurrences of denoting concepts seem to be their commonplace occurrences (from the point of view of the use of denoting phrases in ordinary English), Russell is justified in regarding entity occurrences as questionable—even if his argument for the result was confused.

Moreover, Russell came to draw the proper consequence of the problem of structure. The use of a single letter to represent a meaning occurrence of a complex is illegitimate. He writes:

> It is a fallacy to use a single letter to represent an occurrence of a complex as *meaning*, since a single letter will have all entities among its values; moreover, when a *complex* occurs as meaning, its structure is essential to its significance, and a single letter, since it does not symbolize any structure, destroys the significance. ([OF], *Papers* 4: 374)

A single letter acts as a genuine proper name, and thus the designation of the letter would occur "as entity" and not "as meaning". Couple this with Russell's analysis of the constituents of propositions indicated by nominalizing formal implications, and one sees that it is impossible to quantify over meaning occurrences. This point has a direct bearing on Russell's argument against denoting concepts. Since a genuine proper name indicates an entity occurrence, it follows that there are no such names of denoting concepts and "... the meaning cannot be got at except by means of denoting phrases" ([OD], p. 111; *Papers* 4: 421).

Let us, then, suppose with Russell that (given the difficulty of logical form) we should get along without entity occurrences. On such a view, a denoting concept is, as it were, that which occurs as meaning. This makes sense of the following cryptic and much maligned passage of "On Denoting":

> This leads us to say that, when we distinguish meaning and denotation, we must be dealing with the meaning: the meaning has denotation and is a complex, and there is not something other than the meaning, which can be called the complex, and be said to *have* both meaning and denotation. (P. 112; *Papers* 4: 422)

The wording is certainly cumbrous. Russell's point, however, is simple.

If denoting concepts have only meaning occurrences, and what occurs as meaning is really not one entity at all, then denoting concepts *are* their meaning occurrences—as it were. That is, it is improper to speak of a denoting concept as "having" a meaning occurrence and (sometimes) a denotation. Adjusting Russell's passage, we get:

> This leads us to say that, when we distinguish meaning and denotation, we must be dealing with the meaning: the meaning has denotation and is a complex, and there is not something other than the meaning [i.e., something other than the meaning occurrence] which may be called the complex [logical subject] and be said to *have* both a meaning [occurrence] and a denotation.

The point is just to consider what happens if denoting concepts can only occur "as meaning".

Recall that the *Principles* maintains that denoting is a logical relation explained by the following rule:

> All denoting concepts ... are derived from class-concepts; and a is a class concept when "x is an a" is a propositional function. The denoting concepts associated with a will not denote anything when and only when "x is an a" is false for all values of x. (*PoM*, p. 74)

(Of course, a uniqueness condition will need to be added for denoting concepts indicated by definite descriptions.) The law says that a denoting concept, say 'all men', denotes Socrates (among others) only in so far as Socrates has the property of being a man. But as we have seen, Socrates has this property only in so far as Socrates occurs "as entity" in the proposition, 'Socrates is human'. To exemplify a property is to occur as entity in a true proposition predicating the property. Now let us apply the logical relationship to the situation where denoting concepts are themselves to be denoted. The denoting concept indicated by the phrase "the meaning of "any man" ", denotes 'any man' only if 'any man' occurs as entity in a true proposition predicating the property of being a meaning of "any man". The *Principles*' logical relationship (law) of denoting requires entity occurrence. If there are no such occurrences of denoting concepts, the law is forfeit.

It should be noted that this is not Cassin's "falling-through" argument, that "when a denoting concept occurs in a proposition its denotation occurs" (Cassin *1971*, p. 270). Nor is it Hylton's "principle of truth-value dependence". On this principle,

> ... for a proposition containing a denoting concept to be about some other entity is for the truth-value of that proposition to be dependent upon the truth-value of the proposition obtained from it by replacing the denoting concept by the denoted entity. (Hylton *1990*, p. 251)

Consider the proposition,

'the author of *Waverley* equals Scott'.

In Hylton's view, its truth-value would be dependent upon the truth-value of

'Scott equals Scott'.

Hylton's principle is mistaken, however. The principle violates the structural difference between occurring "as meaning" and occurring "as entity". One cannot substitute Scott for 'the author of *Waverley*' in its meaning occurrence any more than one can substitute Scott for 'Humanity' in 'Socrates is human'. Both violate the structure of the propositions in question.

The present argument is simply that the law of denoting of the *Principles* requires that denoting concepts are capable of the twofold occurrence that concepts enjoy. The logical relationship of denoting has the following consequence:

> Denoting concepts for other denoting concepts are impossible unless denoting concepts have entity occurrences.

The logical relationship requires that a denoting concept denotes a given entity in virtue of the entity having the property (or properties) which form the class concept from which the denoting concept is formed. But an entity has a property in virtue of its occurring "as entity" in a proposition predicating the property. If denoting concepts are not capable of entity occurrences, then they cannot be denoted by other denoting concepts.

If denoting concepts cannot occur as entity, then the fundamental doctrine of the *Principles*, "Whatever is, is one", will be lost and with it

will go its law of denoting. Nonetheless, Russell entertains the possibility of modifying the fundamental principles and formulating a new law of denoting exempting denoting concepts. But he quickly discovers that there is no way modify the logical relationship of denoting to accommodate the view that there are only meaning occurrences of denoting concepts. If there are only meaning occurrences of denoting concepts, Russell writes, "... this only makes our difficulty in speaking of meanings more evident" ([OD], p. 112; *Papers* 4: 422). We cannot allow that one denoting concept *C*, say 'the meaning of "all men" ', can denote another *D*, say 'all men', in virtue of *D*'s occurrence "as meaning" in the true proposition 'all men is a meaning of "all men" '. The proposition is about all men, and not about the denoting concept 'all men'. So suppose *C* denotes *D* in virtue of the presence of a meaning occurrence of a third denoting concept *E*, denoting *D*. Surely this won't do either; it is either circular (if *E* is just *C* itself), or it embarks one on a vicious regress of denoting concepts which are supposed to denote *D*. In virtue of what, then, does *C* denote *D*? We are left with no explanation, and the relationship remains "wholly mysterious" ([OD], p. 113; *Papers* 4: 422).

There is no hope in a theory of propositions that has only meaning occurrences of denoting concepts. Russell needs both meaning and entity occurrences of denoting concepts, but no theory of logical form could ground their structural differences.

7. AN ARGUMENT AGAINST FREGE'S SINN?

Russell's argument depends upon the fundamental principle of the *Principles* that whatever is, is a logical subject (i.e., can occur "as term of a proposition"). An entity has a property or stands in a relation in virtue of its occurring as term of a proposition predicating the property or relation. The capacity of concepts, and denoting concepts in particular, to have a two-fold occurrence is essential to this doctrine. Frege, however, models predication in terms of the mathematical notion of a function, and not in terms of occurrence "as term" in a true proposition (or, if you like, an obtaining state of affairs). Being an "argument" to a Fregean function is completely unlike Russell's notion of occurring "as entity" in a proposition. A function yields a value for an argument. The value is not a whole composed of function and argument (occurring as entity). For this reason, a Fregean *Sinn* can be argument to a function without the least difficulty.

Russell used the expression "entity" synonymously with the expression "logical subject", maintaining that the very notion of an "entity" lies in the capacity to occur "as term of a proposition". We saw that this view grounded Russell's doctrine that any calculus for logic must adopt one style of variables (i.e., "entity variables"). This is wholly alien to Frege. To be sure, Frege maintains that any calculus for logic must adopt "logical object" variables in so far as all logical objects are on a par. But Frege regards functions as entities which are not objects, and thereby introduces special function variables. Accordingly, unlike Russell, Frege can maintain that a *Sinn* always occurs (as it were) "as concept" ("as meaning").

Now Frege does hold that a *Sinn* occurs in a *Gedanke* (the sense of a declarative sentence) as a part. A *Gedanke* is a whole composed of the senses of the meaningful parts of a sentence. But this occurrence is always "as concept" and never "as term". Indeed, this is so even in a "referentially shifted" context, Frege held that the context of an intentional verb produces a referential shift from customary reference to customary sense. Thus in

> "George wondered whether the author of *Waverley* wrote *Ivanhoe*",

the reference of "the author of *Waverley*" is not Scott but is shifted to the sense of "the author of *Waverley*". Nonetheless, the *Gedanke* of the whole sentence does not contain this sense, or any other sense "as term". Rather it contains, in a way analogous to Russell's notion of an occurrence as concept, a sense referring to the sense of "the author of *Waverley*".

Frege allows only one sort of an occurrence in a *Gedanke*—viz., occurrence "as concept" ("as meaning"). A *Sinn* always occurs "as meaning". This spelled ruin for Russell's denoting concepts. But it cannot touch Frege simply because he adopts functionality rather than predication as primitive.[17]

[17] In Appendix A of *PoM*, Russell criticized Frege's adoption of functionality and 'the True' and 'the False'. Russell may have presumed that interested readers of "On Denoting" would recover this criticism. His argument against denoting cannot score against the historical Frege, but it would be successful if Frege's adoption of functionality were dropped in favour of the view that the *Bedeutung* of a declarative sentence is not a truth-value but a Russellian proposition.

So it is Frege's adoption of functionality that saves him. The very thing that saves him, however, is what prevents him from arriving at Russell's new theory of definite descriptions. Frege's *Begriffsschrift* was well ahead of Russell in its forthright rejection as logically insignificant the fact that "all men" is the grammatical subject of "all men are mortal". Engulfed for a time by the quagmire produced by the theory of denoting concepts, Russell finally broke free in 1905, joining ranks with Frege. But Russell was able to advance further than Frege in this quarter, and necessary to this is the fact that he introduced variables into logic without adopting functionality as primitive.

To see this, let us briefly examine a counter-argument. It is sometimes argued that Frege uncovered the "logical fact" behind Russell's theory of definite descriptions. Tichý (*1988*), for instance, notes that in his *Grundgesetze*, Frege takes the following[18] as axioms governing his primitive function symbol "$\backslash z$" for "the so & so":

(iv) $\vdash \backslash\acute{z}(z=a) = a.$

(*) $$\begin{array}{l} \vdash\!\top\ \backslash a = a \\ \quad\llcorner \overset{x}{\smile}\ a = \acute{z}(z=x). \end{array}$$

Tichý then observes that the following is provable:

$$\begin{array}{l} \vdash\!\top\ g(\backslash\acute{z}fz) = \top\overset{x}{\smile}\top\ gx \\ \quad\mid \qquad\qquad\qquad\qquad \llcorner \overset{y}{\smile}\ fy = (x=y) \\ \quad\llcorner\!\top\ g(\acute{z}fz) \end{array}$$

This he proclaims is the essential discovery. Of course, one might object to the antecedent clause. The antecedent is needed to assure the identity. It may be that the value of g with argument z is *the True* if z is a class, and is *the False* otherwise. Let f be, say, a function which assigns an object *the True* if it is winged and a horse, and otherwise assigns *the False*. The identity fails, for $g(\backslash\acute{z}fz)$ will be *the True*. The class of all winged horses is the empty class. Tichý glosses the need for the anteced-

[18] The second is implicit in Frege's *Grundgesetze*.

ent, saying that in the interesting cases, g will not be such as to hold of a class. It was only Frege's rejection of partial functions that demands that the function g assign a value for all arguments—even classes. It is a "category mistake," he says, "to form the statement 'the ϕ is a ψ' where ϕ is a property of individuals and ψ is true, *inter alia*, of the *extension* of ϕ" (Tichý *1988*, p. 122).

Tichý's introduction of the notion of a "category mistake" is but a red herring. Frege adopts a "chosen object" approach—an approach which is necessitated because the language for Frege's system includes "$\backslash a$" as a genuine singular term. Frege could have avoided (*) for,

$$(**)\quad \vdash\!\!\top\, \backslash a = \blacktriangledown$$
$$\qquad\quad \llcorner\!\!-\, x \top\, a = \grave{z}(z=x).$$

where $\blacktriangledown$ is whatever "category" Tichý wishes. Again, Tichý must face the fact that in some cases the identity fails: the class $\grave{z}fz$ (of winged horses) is not a singleton. Let g be a function that takes objects to *the True* just when they are short; let $\blacktriangledown$ be Napoleon Bonaparte. Then $g(\backslash\grave{z}fz)$, that is $g(\blacktriangledown)$, is *the True*. There is no "category mistake" here!

Frege is committed to a "chosen object view" precisely because he takes "$\backslash a$" as a singular term. And it could not have been otherwise. In Frege's "function calculus", "fx" is not a formula but a name for *the* value of the function with argument x. With functionality as a primitive, Frege could not hope to analyze definite descriptive phrases. Russell, on the other hand, has a predicate calculus (of sorts) and boldly proclaims that "ι", unlike Frege's "$\backslash z$" is not a term-forming operator (so that when one puts a term for the position of "z" one gets a term). In "On Fundamentals" (completed in June of 1905) Russell doesn't officially write out his contextual definition, though he applies it several times.[19] In the 22 December 1905 manuscript "On Substitution," however, we find ([OS], folio 4):

$$\phi!\{(\iota x)(\psi!x)\}\ .=:\ (\exists b):\psi!x\ .\!=_x.\ x=b:\ \phi!b\quad \mathrm{Df}^{20}$$

[19] It is applied to the Propositional Liar ([OF], *Papers* 4: 401), to the introduction of class terms ([OF], 4: 397), and there is a discussion of scope as well (*ibid.*).

[20] The shriek is not relevant to the present discussion.

From this he gets an equivalence. The reason he needs no antecedent clause is precisely because his expression "$(\iota x)(\phi x)$" is *not* a singular term of his language. In "On Fundamentals", Russell wrote:

On this view we shall not introduce ι'u at all but put

$$\phi`(\iota`u)\ .=:\ (\exists y):y\varepsilon u : z\varepsilon u\ .\supset_z.\ z=y:\ \phi`u)\quad \mathrm{Df}$$

This defines all propositions about $\iota`u$, which is all we need. But now $\phi`(\iota`u)$ is a bad symbol: we shall have to substitute (say)

$$(\phi\,\iota)`u.$$

On this view, "the author of *Waverley*" has no significance by itself, but propositions in which it occurs have significance. Thus in regard to denoting phrases of this sort, the question of meaning and denotation ceases to exist. (*Papers* 4: 384)

Russell adopts notation from Peano, of course. Like Frege's function symbol "$\backslash z$", Peano had "$\urcorner\, z$" (the inverse of the singleton functor) and axioms similar to Frege's governing it. When the class term "u" names a singleton whose sole member is a, the function, , yields $\urcorner\, u = a$; otherwise, it goes undefined (Peano *1900*, p. 351). (Similarly, for the class term "$x\ni\phi x$" Peano has: $\urcorner\,(x\ni\phi x) = a$, when the class $x\ni\phi x$ is a singleton whose sole member is a.) Russell adopts no function $\urcorner$ and no function sign "$\urcorner\, z$". Russell's sign "ι" is an inseparable part of "$\theta(\iota u)$" and that is why Russell wishes to alter the notation. Indeed, there being no such function, "ι" does not stand for an operator and does not attach to (class) terms like "$x\ni\theta x$" to form a term. Russell's does modify Peano's notation, putting "$\iota x\theta x$" and "$\phi(\iota x\theta x)$", and "ι" is just as inseparable from "$\iota x\theta x$" as "$\iota x\theta x$" is an inseparable part of "$\phi(\iota x\theta x)$". As a result, Russell avoids the "chosen object" approach with his theory of "incomplete symbols" contextually defined. Russell now captures mathematical functionality

$$f(x) = y$$

with the contextual definition of,

$$(\iota z)(xfz) = y.$$

Here there is no syntactic and semantic unit "$(\iota z)(xfz)$" at all. This was Russell's unique advance over both Frege and Péano.

8. THE VARIABLE

Russell's discovery that denoting phrases cannot be treated as syntactic and semantic units is very important. Historically, it shows that 1905 marks Russell's 180-degree turnabout with respect to the problem of the variables of quantification. In "On Denoting", Russell abandoned all hope of giving a metaphysical theory of the use of single letters as variables. As we saw, Russell had felt constrained to offer such a theory because of his commitment to general propositions. He needed an account of the constituents of those propositions named by nominalizations of formulas containing single letters used as variables; and he hoped to find such an account by employing the theory of denoting concepts and the notion of the substitution of entities. If there are genuine propositions, what are their constituents? In "On Fundamentals" Russell momentarily demurred: perhaps the denoting concepts 'every thing' and 'any thing' might be kept so as to answer the question as to the constituents of general propositions. But he quickly faces the fact that his argument is conclusive against all denoting concepts ([OF], *Papers* 4: 387). With the abandonment of denoting concepts, Russell abandoned the whole project of offering a metaphysical theory of the use of variables. The assignment of a variable (its "determination" in Russell's words) is taken to be primitive and wholly unanalyzable. This point of view is not unlike the modern conception which regards the assignment of the variables of the object language of a theory as a semantic issue of the interpretation of the language over a domain. The way a variable gets assigned to this or that value is something that comes outside any statements made within the language of the theory itself.

Unfortunately, this radical change has been missed. One reason is that Russell maintained an ontology of general propositions after 1905, albeit sporadically. So it is thought that Russell must have intended some new ontological account of the use of variables. Moreover, in "On Denoting", Russell wrote:

> I take the notion of the *variable* as fundamental; I use "$C(x)$" to mean a proposition in which x is a constituent, where x, the variable, is essentially and wholly undetermined. Then we can consider the two notions "$C(x)$ is always true" and "$C(x)$ is sometimes true". ([OD], p. 104; *Papers* 4: 416)

In an adjoining footnote Russell adds that 'C(x)' is a propositional function. This suggests that somehow the variable was to be ontologized as a part of an entity called a "propositional function".[21]

But it is quite clear that this interpretation is mistaken. First, observe that if Russell had embraced an ontology of propositional functions, he could have solved the problem of the difference in structure between meaning and entity occurrences of denoting concepts! Cocchiarella (*1989*) has shown the way. A denoting concept such as 'the author of *Waverley*' can be represented as the property a property P has if and only if P is a property exemplified by the unique entity who authored *Waverley*. That is, there is a property of properties F whose exemplification conditions are such that

$$(P)(F(P) \equiv (\exists x)((y)(\text{authored } \textit{Waverley}(y) \equiv y{=}x)\ .\&.\ P(x)).$$

Thus letting G be the property of being equal to Scott, the structure of the proposition,

'the author of *Waverley* equals Scott'

is just $F(G)$. We see that here 'the author of *Waverley*' occurs "as meaning" (i.e., predicatively). On the other hand, putting D for the property of being a denoting concept, the structure of the proposition

' 'the author of *Waverley*' is a denoting concept'

is represented by $D(F)$. As we see, represented as a property of properties, the twofold occurrence of the denoting concept can be made intelligible. But this escape from the Gray's *Elegy* argument of "On Denoting" presupposes that one has a solution of the Russell paradox of predication. Obviously, Russell had no such solution to rely upon. Indeed, it was the denoting concepts and the notion of substitution that Russell had hoped to show a way to the solution of the paradoxes.

[21] See Hylton *1991*.

In fact, at the time of "On Denoting" Russell felt that he had found the way to a genuine solution of the paradoxes (of classes and predication) by avoiding an ontological commitment to propositional functions or classes. His new 1905 theory of definite and indefinite descriptions bore immediate fruit in his so-called substitutional theory.[22] With the ontological notion of substitution distinguished from the new unanalyzable (semantic) notion of the variable (and its "determination") Russell was able to show that a type-stratified language of predicate variables can be proxied in a type-free calculus for the pure substitutional logic of propositions.[23]

The problem of the legitimacy of quadratic forms, i.e., the problem of discovering when it is legitimate to proceed from a formula $A\mu$ by introducing variables "ϕ" and "x" and bind them independently in "ϕx", is solved in the substitutional theory. The theory shows how to proxy, in a calculus with only entity variables, a type-stratified language with nominalized predicates and bindable predicate variables—a calculus, as it were, of "propositional functions". The primitive new wff, "$p/a;b!q$", is adopted for the notion of "substitution". A convenient reading is:

"q is exactly like p except containing b at every occurrence of a in p".

In "On Substitution" Russell adds ([OS], p. 4):

$$p/x;a =_{\mathrm{df}} (\iota q)(p/x;a!q).$$

Returning to the problem at hand and using "$\phi^{(\mathrm{O})}$" as a variable for an attribute of individuals and "x^{O}" as a variable for an individual, the expression:

$$\text{“}(\forall\phi^{(\mathrm{O})})(\forall x^{\mathrm{O}})\,(\phi^{(\mathrm{O})}(x^{\mathrm{O}}) \supset \phi^{(\mathrm{O})}(x^{\mathrm{O}}))\text{”}$$

will be proxied by means of

[22] The principal manuscript for the substitutional theory is "On Substitution", dated 22 December 1905. There is also a hint of Russell's breakthrough in the first footnote of his November 1905 paper, "On the Relation of Mathematics to Logic" ([RML], p. 261).

[23] See his 14 March letter to Jourdain, reprinted in Grattan-Guinness *1977*, p. 79.

$$\text{“}(p)(a)(p/a;x \supset p/a;x)\text{”}.$$

The legitimate use of predicate variables and "quadratic forms" is revealed via the proxy. Using "$\phi^{((\mathrm{O}))}$" as a variable for attributes of attributes of individuals, the next type is:

$$\text{“}(\forall\phi^{((\mathrm{O}))})(\forall\psi^{(\mathrm{O})})(\phi^{((\mathrm{O}))}(\psi^{(\mathrm{O})}) \supset \phi^{((\mathrm{O}))}(\psi^{(\mathrm{O})}))\text{”}.$$

This is proxied by

$$(q, p, a)(r, c)(q/p, a;r, c \supset q/p, a;r, c)\text{”}.$$

Here Russell puts:

$$q/p, a;r, c =_{\mathrm{df}} (\iota t)(q/p, a;r, c!t),$$

and reads "$q/p, a;r, c!t$" as saying:

"t is exactly like q except for containing the entity r at every occurrence of p and containing c at every occurrence of a in t."

Such a multiple substitution is defined in terms of a carefully crafted succession of single substitutions. The process continues as one ascends types. But Russell's paradox is solved. An expression such as "$\phi(\phi)$" is inexpressible in the formal grammar of the substitutional theory. Types become part of logical form.

Quite clearly, Russell did not conceive of "On Denoting" as ontologizing the variable by means of a theory of propositional functions. When Russell says that '$C(x)$' is a "propositional function," he simply means that he is using the expression "$C(x)$" schematically for a formula containing the variable "x" free. The reason Russell wrote "'$C(x)$' is always true" is that he wished to offer the truth conditions for the expression "$(x)(Cx)$".

But what of the problem of the constituents of general propositions? Well, Russell simply had no answer to the problem. In fact, Russell's own intellectual honesty affords a proof. In a letter to Moore of 25 October 1905, he responded to precisely this question:

I only profess to reduce the problem of denoting to the problem of the variable. This latter is horribly difficult, and there seem equally strong objections to all the views I have been able to think of.[25]

In fact, it would not be long until he felt himself ready to abandon general propositions altogether because they give rise to certain paradoxes esoteric to substitution.[26]

9. DENOTING PHRASES AS "LOGICAL UNITS"

As we see, a proper understanding of Russell's 1905 view of variables is central to grasping the historical development of *Principia Mathematica*. But Russell's abandonment of denoting is also of importance for assessing recent theories which, in an attempt to parallel the surface grammar of natural language, hope to regard denoting phrases as syntactic and semantic units. Evans (*1977*, *1982*) has argued that the phenomena of anaphor in natural language provides conclusive evidence against any theory which views definite descriptions as logical units. Consider the following example:

(I) Russell bought some hens and Whitehead vaccinated them.

English grammar appears to make "them" anaphoric on "some hens." A post-1905 transcription, however, would put the following as its logical form:

(IIa) $(\exists x)(\mathrm{Hen}(x)\ \&\ \mathrm{Bought}(R, x))\ \&\ (\forall x)(\mathrm{Hen}(x)\ \&\ \mathrm{Bought}(R, x)\ .\supset.\ \mathrm{Vaccinated}(W, x))$.

Here we see that the apparent anaphora of the surface grammar is lost in the logical form. One might think to capture the anaphora by making "them" a variable bound to "some hens":

[25] Russell to G. E. Moore, 25 October 1905; copy in RAI 710.053032.

[26] See Russell's "On 'Insolubilia' and Their Solution by Symbolic Logic" [InS]; and Landini *1989*.

(IIb) $(\exists x)(\mathrm{Hen}(x)\ \&\ \mathrm{Bought}(R, x))\ \&\ (\forall x)(\mathrm{Hen}(x)\ \&\ \mathrm{Bought}(R, x)\ .\supset.\ \mathrm{Vaccinated}(W, x))$.

This, however, would be true if Whitehead vaccinated only one among the several hens Russell bought. Evans concludes that phenomena of anaphora in natural language shows that Russell was quite right in rejecting the view that denoting phrases are syntactic and semantic units—albeit for the wrong reasons.

Recently, it has been shown that anaphoric features of natural language can be preserved in the face of such apparent counterexamples. One can treat "them" in (I) as functioning as a plural description. We have:

(III) Russell bought some hens and Whitehead vaccinated the hens that Russell bought.

Following this approach, Neale (*1990*) maintains that the syntactic unity of "some hens" and "the hens Russell bought" is preserved in:

(IV) [some$_1$ hens x_1](Russell bought x_1) & [the$_1$ hens x_1Russell bought x_1](Whitehead vaccinated x_1).

Here "the hens Russell bought" is anaphorically tied to "some hens", and this, Neale says, preserves the surface structures of English. Neale maintains that the formal apparatus of Russell's 1905 theory of definite descriptions can be preserved. In the sentence,

(V) The author of *Waverley* is Scott,

one can simply put:

(VI) [Some author x, uniquely wrote *Waverley* x](x equals Scott).

This, he contends, captures "the author of *Waverley*" as a logical unit.

Of course, it was Montague who pioneered all this; and both Neale and Cocchiarella are working from within the tradition which he inaugurated. But it is far from clear how Neale's construction preserved denoting phrases as logical units (i.e., as syntactic and semantic units).

The trouble is that Neale uses variables. This makes Neale's (IV) but a notational variant of (IIa), and instead of Neale's (VI) we may as well put:

$$(\exists x)((y)(\text{Author of } \textit{Waverley}(y) \equiv y{=}x) \ .\&.\ x{=}\text{Scott}).$$

Indeed, once variables are introduced, denoting phrases lose their status as syntactic and semantic units altogether.

Denoting phrases act as syntactic and semantic units only in so far as variables are not employed. In "Every man is mortal", the phrase "every man" performs without benefit of variables. This is what is distinctive about denoting phrases in English. In order to capture the syntactic and semantic unity of denoting phrases, one must return to Russell's theory of denoting concepts. Any return, however, must confront his "Gray's *Elegy*" argument—viz., the problem of the logical form of occurrences of denoting concepts. Cocchiarella shows precisely how to do this. His constructions are framed from within a type-free second-order calculus of attributes. Attributes (properties and relations in intension) are not structured entities, so the variables used in stating their exemplification conditions are not relevant to their being as intensional entities. By representing Russell's denoting concepts as properties of properties, Cocchiarella can do justice to the fact that denoting phrases in English act as syntactic and semantic units. On Cocchiarella's theory,

$$\text{“}(\exists x\text{Hen})(\text{Bought}(R, x))\text{”}$$

is not just a notational variant of

$$\text{“}(\exists x)(\text{Hen}(x) \ \&\ \text{Bought}(R, x)).\text{”}$$

Rather, its logical form is '$F(G)$' where F is such that:

$$(P)(F(P) \equiv (\exists x)(\text{Hen}(x) \ \&\ Px)),$$

and G is the property of being a thing bought by Russell. Similarly, the logical form of (V) is '$F(G)$' where F is now such that:

$$(P)(F(P) \equiv (\exists x)((y)(\text{Author}(y, W) \equiv y{=}x) \ \&\ P(x))),$$

and G is the property of being equal to Scott. Clearly in Cocchiarella's construction, variables are not employed as a part of the logical form. The denoting phrases are genuinely syntactic and semantic units.

In light of this, we see that Cocchiarella has a genuine reply to Evans' attempt to vindicate Russell's views against denoting concepts. However, once we see the true force of Russell's Gray's *Elegy* argument, we see that Evans should not be so quick to dismiss Russell's own reasons against theories that take denoting phrases to be logical units. To genuinely preserve denoting phrases, one must be able to distinguish the logical form of propositions in which denoting concepts occur "as meaning" (predicatively) as opposed to those in which they occur "as entity". Cocchiarella has shown how to do this. But his answer—indeed, any viable answer—requires a solution to Russell's paradox of predication. Though Cocchiarella's theory of attributes is very attractive, it must be admitted that it falls short of a genuine solution of the paradoxes. Accordingly, Russell was right after all; and, Evans notwithstanding, right for the right reasons.

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