FUNCTIONS OR PROPOSITIONAL FUNCTIONS?

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[7] In, respectively, *Pacifism in Britain* and *Semi-Detached Idealists: the British Peace Movement and International Relations* (Oxford: Oxford U. P., 2000).

[8] See Monk 2: Chap. 13.

Gottlob Frege is widely recognized as one of the chief progenitors of mathematical logic and philosophy of language — indeed, the importance attributed to Frege's innovations is such that Michael Dummett, when reflecting on §62 of Frege's *Die Grundlagen der Arithmetik*, feels compelled to remark that the paragraph therein is "arguably the most pregnant philosophical paragraph ever written."[1] Whether or not one agrees with Dummett on this point, the magnitude of Frege's influence on the current philosophical landscape is incontestable. Frege revolutionized the then dominant Aristotelian conception of logic, introducing a formal language now recognized as the predicate calculus. Central to this end were Frege's insights on quantification, the notation that expressed it, the logicist project, and the extension of mathematical notions like function and argument to natural language. The long-awaited *Cambridge Companion to Frege* is a compendium of Fregean scholarship that rigorously explores these and similar topics; editors Thomas Ricketts and Michael Potter have compiled a comprehensive collection of fourteen essays that individually provide focused appraisals of a number of Frege's most substantial insights. On the advice of this journal's editor, I have limited my review to the connections (or rather dissimilarities) that exist between Frege and Russell, relying exclusively on Peter Hylton's contribution to the collection, "Frege and Russell". Following a brief consideration of the volume as a whole, I move immediately into a detailed explication of Hylton's analysis.

The volume includes among its goals the following two aims: (i) to provide, for advanced students and specialists, a conspectus of recent interpretation of Frege, and (ii) to dispel the intimidation felt by readers who confront a difficult thinker.[2] I single out these ambitions because they most clearly represent the volume's strengths and shortcomings. Enthusiasts of Frege will not be disappointed by the multitude of rigorously and technically written essays, essays that contribute substantially and originally to the collective understanding of Frege and his work. For instance, Thomas Ricketts provides a thoughtful and thorough analysis—an analysis that moves well beyond mere explication—of Frege's concept/object distinction, in particular its relevance for Frege's Context Principle and logicism more generally. Similarly, Richard Heck addresses a number of central yet largely unexplored issues pertaining to the overall significance of

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[1] *Frege: Philosophy of Mathematics* (Cambridge, MA: Harvard U. P., 1991), p. 111. The paragraph contains an expression of Frege's Context Principle and the need for Hume's Principle as a criterion of identity in the definition of number.

[2] There are in fact five stated goals. The others are: (iii) to serve as a reference work for students and non-specialists, (iv) to provide a route into the study of Frege historically and to any relevant contemporary concerns in the philosophies of logic, language, mathematics, and mind, and (v) to provide, for new readers, the most convenient detailed guide to Frege currently available.

Frege's project; indeed, the great majority of essays in the volume are nothing less than cutting-edge assessments of familiar Fregean topics. For these very reasons, *The Cambridge Companion to Frege* is an invaluable and necessary resource for any serious student of Frege. The *Companion* therefore excels at (i). Yet, the inevitable downside of such innovative scholarship is that, for those new to Frege, the essays may increase one's intimidation rather than dispel it. The introductory essays supplied by Michael Potter and Joan Weiner, while well written and insightful, are in the main insufficient to prepare the novice for the essays that follow. To some degree this is to be expected, for Frege himself was a highly technical and difficult thinker, and thus accomplishing (ii) is perhaps an insurmountably difficult task.[3] In the end, this criticism should do little to deter committed students of and specialists in Frege from seriously engaging with this *Companion*.

Hylton's contribution to the collection (pp. 506–49) involves a comparative analysis of the underlying metaphysics of Frege and Russell, specifically as it relates to and engenders their divergent logical systems.[4] From the start, Hylton notes that his concern is with the period spanning 1900 to 1920—namely, the period flanked by Russell's break with idealism and his shift toward pragmatism and behaviourism—and that his attention is directed toward the *dissimilarities* between Frege and Russell. In light of this, Hylton clarifies that his intentions are not to downplay the similarities between the two thinkers; rather, "it is because their views are in some ways so similar, and the pairing so natural, that differences between them are of great interest" (p. 510). The central conclusion of the essay is Hylton's identification of a fundamental difference between Frege and Russell: Frege takes *functions* as primitive whereas Russell takes *propositional functions* as primitive. The bulk of the essay (§§1–6) contains a lucid interpretation of how and why this difference came about. What follows is an explication of the central steps in Hylton's argument.

The Fregean distinction between *Sinn* and *Bedeutung*—here translated as *sense* and *reference*, respectively—is described by Hylton (in his §5) as having two primary kinds of motivation: first, there is the issue of empty names, the fact that we understand names even in instances where they do not refer (e.g. the name "Pegasus"). Second, there are cases of co-referring expressions in which the relevant expressions demonstrate different semantic roles (e.g. the expressions

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[3] I strongly encourage that non-specialists augment *The Cambridge Companion to Frege* with Joan Weiner's *Frege Explained: from Arithmetic to Analytic Philosophy* (Chicago: Open Court, 2004), one of the best available introductory texts on Frege.

[4] The essay in fact was available as early as 2005 in Hylton's *Propositions, Functions, and Analysis: Selected Essays on Russell's Philosophy* (New York, NY: Oxford U. P., 2005). The editors of *The Cambridge Companion to Frege* partially account for this in their Preface, mentioning that the volume has been "many years in the making" (p. xiii).

"the husband of Xanthippe" and "the teacher of Plato" seem to have distinct semantic roles or meanings, even though they both refer to Socrates). For primarily these reasons, Frege denies that the meaning of an expression is *solely* the object it refers to; instead, the name Pegasus or the expression "the teacher of Plato" express *Sinne*, or senses, and it is therefore *Sinn* that constitutes the content of our expressions. Hylton portrays Russell's rejection of Fregean *Sinne* as the product of Russell's direct realism—that is to say, the "insistence on a direct and unmediated relation between the mind and the known object and the idea that propositions paradigmatically contain the entities they are about."[5] The example provided is the widely recognized Mont Blanc illustration, which stems from the correspondence between Frege and Russell in late 1904. Frege incidentally suggests that, in the thought expressed by the sentence "Mont Blanc is more than 4,000 metres high", the actual Alpine mountain does not in fact occur as a component part of the thought (or proposition); instead, the *Sinn* of the expression "Mont Blanc" occurs in the thought, and it is this that in turn picks out the actual mountain.[6] Hylton convincingly argues that Russell here denies Fregean *Sinne* for epistemological reasons. Russell's specific concern is what Hylton labels the *in-virtue-of* problem: "how, in virtue of grasping that entity [that *Sinn*], do we know something about the mountain, which is altogether distinct from it?" (p. 516). In other words, *how*, precisely, does a sense correspond to its referent? Since Russell cannot discern any adequate explanation for this, he assumes that "the presence of a name in a sentence implies, at least in paradigmatic cases, that the sentence expresses a proposition which contains the named object" (p. 515). As such, Russell takes Mont Blanc, the mountain with all its snowfields, actually to be contained within the proposition. Hylton contends that Russell's direct realism—*vis-à-vis* his early theory of acquaintance—is a view that Russell always instinctively drifted toward, and, in fact, this contention plays a significant role in Hylton's reading of Russell's acceptance of the theory of descriptions in 1905.

Prior to discussing that theory however, Hylton considers (in §3) various problems facing Russell's direct realism, and then turns his analytic lens to the theory of denoting concepts conscripted to solve (or soften) them. Again, Hylton mentions two particular problems: first, there is the difficulty of non-existent concreta (e.g. "Socrates" or "Pegasus"). This difficulty seems to parallel the first motivation, mentioned above, for Frege's theory of sense. (Instead of "non-

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[5] Hylton, "The Theory of Descriptions", in *The Cambridge Companion to Bertrand Russell*, ed. Nicholas Griffin (Cambridge: Cambridge U. P., 2004), p. 209; reprinted in *Propositions, Functions, and Analysis*, p. 192.

[6] Frege, *Philosophical and Mathematical Correspondence*, ed. Gottfried Gabriel *et al.* (Oxford: Blackwell, 1980), p. 163 and *Wissenschaftlicher Briefwechsel*, ed. Hans Hermes *et al.* (Hamburg: Meiner, 1976), p. 245.

existent concreta", Hylton uses the terminology "empty names" in describing the first motivation for Frege's theory of sense; I cannot see a substantial difference between the two.) Second, and more substantially, there is the issue of generality (e.g. sentences that express propositions like "Every natural number is either odd or even"). If, as Russell's direct realism demands, the referent of some name or expression is actually contained in the proposition, then both non-existent concreta and generality appear to create insurmountable problems for his direct realism; for in the case of non-existent concreta there is nothing that could be contained in the proposition, and in the case of generality, there appear to be infinitely complex propositions. These difficulties led Russell to adopt his theory of denoting concepts, which, instead of problematically granting that every natural number is a constituent of the proposition "Every natural number is either odd or even", posits a *denoting concept* as contained in the proposition, which concept in turn denotes the natural numbers. As such, definite and indefinite descriptions under the theory of denoting concepts are *indirectly* about their referents. Hylton observes that, like Frege's theory of sense, there is an in-virtue-of problem here, for how is it that a denoting concept, in virtue of its being contained in a proposition, is about that which it denotes? Thus put, we might make explicit two features of the theory of denoting concepts: (i) it posits a form of indirect aboutness (via denoting concepts) and (ii) it suffers from an in-virtue-of problem. The process here is indirect because the referent is not contained in the proposition (a denoting concept is); and, moreover, there is an in-virtue-of problem, because it is not clear how a denoting concept succeeds in denoting at all. As Hylton observes, "[t]o this question Russell has no answer: the relation of denoting is simply asserted to have that effect" (p. 521). One might wonder how it is that Russell could be satisfied with this, particularly given the nature of his rejection of Fregean *Sinne*, in the Mont Blanc illustration. Nevertheless, as we shall see, in 1905 Russell devised his theory of descriptions, and therefore in the end avoided the issue altogether.

Hylton, in §§4–5, compares the theory of denoting with the theory of descriptions, and explains how the latter avoids any commitment to *Sinn*. The attractiveness of the theory of descriptions, for Russell, as depicted by Hylton, consists in the fact that such a theory preserves (i) but avoids (ii). Recall that under the theory of denoting concepts, a sentence of the form "The President of the USA in 2000 was a Democrat" contains a denoting concept corresponding to the definite description, "the President of the USA in 2000". Since the denoting concept contained in the proposition is said to denote the referent of this description—namely, President Clinton—and therefore since President Clinton is not *himself* contained in the proposition, the proposition is said to be *indirectly* about him. But, as noted, there is an in-virtue-of problem here: the way in which the denoting concept succeeds in being about President Clinton is mysterious and unexplained. The theory of descriptions, in contrast, manages

to preserve indirect aboutness while evading the in-virtue-of problem. Under the theory of descriptions, the sentence "The President of the USA in 2000 is a Democrat" expresses a proposition of the following logical form:

(1) $(\exists x)[Px \;\&\; (\forall y)(Py \supset x = y) \;\&\; Dx]$

which reads, "There is an object x such that x served as President of the USA in 2000 and x was a Democrat, and for every object y, if y served as President of the USA in 2000 then y is identical to x" (*cf.* p. 525). Nowhere in the above formulation does President Clinton himself appear; it is only in virtue of the fact that these properties are *only* exemplified by President Clinton does the above formulation refer to President Clinton. Thus stated, we see at once how the theory of descriptions preserves indirect aboutness and yet avoids the in-virtue-of problem. Hylton writes:

> The sentence … above is certainly about President Clinton. As analysed, however, it expresses a proposition which does not contain that man; it is *indirectly* about him. So one might think that here too, as in the theory of denoting, there is a violation of Russell's direct realism. But in fact this is not so: here there is no in-virtue-of problem. Here the idea of indirect aboutness does not rely on a mysterious relation of denoting…. The sentence is about Clinton because it contains a predicate, "served as President of the USA in 2000", which holds of him and of no one else.   (P. 525)

Having depicted the superiority of the theory of descriptions in relation to the theory of denoting, Hylton shows how the theory of descriptions avoids any commitment to Fregean *Sinne*. This follows from Russell's distinction between apparent (or descriptive) names and logically proper names: if we designate the apparent or descriptive name "the President of the USA in 2000" as ϕ, and x as contained in (1) as a logically proper name, then it becomes evident that ϕ, though grammatically the subject of the sentence "The President of the USA in 2000 is a Democrat", is not the subject of—in fact, does not even occur in—the sentence after analysis. Instead, x constitutes the logical subject. Hylton demonstrates that given Russell's distinction between apparent and logically proper names, our original motivations for *Sinn* are avoided—indeed, empty names and co-referring expressions (i.e. descriptive names) are entirely explained away. Characterizing apparent names as *complex referring expressions* (that is, expressions which "have semantic properties which are not exhausted by their reference"[7]), Hylton portrays Russell's achievement as the removal of complex referring expressions altogether; in fact, "the only genuine referring expressions (for a given speaker) are those which are simple, i.e. lacking semantically significant

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[7] Hylton, "Functions and Propositional Functions in *Principia Mathematica*", in *Propositions, Functions, and Analysis*, p. 129.

structure" (p. 535). Logically proper names are these simple referring expressions, and, as it happens, the *only* sort of referring expression.

Hylton's most substantial contribution occurs in his broader discussion of the discontinuities between the metaphysical and logical systems of Frege and Russell (which occurs primarily in §6). Hylton begins by denying that "analysis" is a mere "pragmatic point of philosophical method"; instead, the "correct method of analysis is the correct way to understand … the fundamental nature of the world" (p. 536). Given this conception of analysis, the specific manner in which Frege and Russell conduct their analysis will have significant ramifications for their metaphysical and ontological views. Hylton argues that Frege's predominant method of analysis—where function and argument are primitive notions—engenders (indeed, Hylton seems to implicitly suggest *entails*) complex referring expressions in our language. The idea seems to be that carving up language according to the function/argument method inevitably gives rise to complex referring expressions, and thus ultimately to the sense/reference distinction. If indeed Hylton intends entailment here, then we have a conditional such that $F \supset S$ (where "F" designates taking function/argument analysis as primitive, and "S" represents the sense/reference dichotomy). Hylton seems to be right about this: we have already seen how sentences like "Socrates is the teacher of Plato", when analyzed according to function/argument, engender the Fregean distinction of sense and reference. Next, Hylton reiterates Russell's rejection of *Sinn* (and thus S in our conditional), thereby entailing the denial of F.

The contrast that I am drawing between Russell and Frege, then, is this. Frege takes the notion of a function as primitive; his doing so commits him to a distinction between sense and reference. But Russell denies that there is such a distinction. He therefore cannot accept the general notion of a function as primitive, and cannot accept Frege's fundamental mode of analysis. (P. 537)

Russell therefore cannot take function/argument analysis as primitive. What then is primitive for Russell? Hylton identifies *propositional functions*, which are also employed in an effort to explain Fregean or non-propositional functions. A propositional function is "the non-linguistic correlate of an open sentence" (p. 543). Put simply, a propositional function is a three-place function represented by $\text{ADD}(x, y, z)$ so that $\text{ADD}(2, 3, 5)$ is a true sentence—in this case, the sentence "The sum of 2 and 3 is 5." Fregean functions are subsequently defined in terms of propositional functions; for instance, the two-place plus function $x + y$ is defined as the object z such that $\text{ADD}(x, y, z)$. The point to observe here, however, is that all complexity is reduced to propositional complexity: there is no complexity beyond the proposition. This allows Russell to account for Fregean functions, yet without the need for anything beyond simple referring expressions. As such, the fundamental difference between Frege and Russell—that is, the difference that holds the most significance for their respective logical

systems—is whether or not they take functions or propositional functions as primitive.

Like most of what Hylton writes, his contribution to the *Cambridge Companion to Frege* is a lucid analysis, one that makes substantive progress in our understanding of two key thinkers. By focusing on Frege and Russell's respective dissimilarities, Hylton has succeeded in identifying some of their more principal contributions. His treatment of the progression of Russell's theoretical views up to and including the theory of descriptions provides a helpful groundwork for those new to Russell, and the subsequent sections (namely, §§6–7) make significant headway in the broader characterization of Russell's logic and metaphysics. More specifically, the discussion that centres on the notion of a propositional function, particularly as it contrasts with Frege's taking function/argument analysis as primitive, is a welcome and original appraisal of the logical systems of both thinkers. Here we witness a fundamental disparity between them. Indeed, Hylton's essay constitutes a model for where Fregean and Russellian scholarship should be heading.